

< The Way of Scientific Research >

Jun Shibata
Asahigaoka High S chool



ABSTRACT

In this essay, vacuum energy will be discussed by the explanation used in experiential intuitions. It was expected to find a breakthrough to the current situation of the research on cosmic acceleration without waiting for new observations to be done. Unfortunately, obstacles were found against this approach. This essay shows the paths taken to solve the problems and how they turned out to be through the discussion with the researchers of cosmic accelerations concluding with points to be overcome to become a valid way.

THIS IS NOT A SCIENTIFIC REPORT. THIS IS MORE LIKE AN ESSAY ON METHODOLOGY.

1. Introduction

“Rearranging a formula”, is one way to deal with the phenomenon of the universe. When it comes to the unknown or at least unaccountable phenomenon, the power of mathematics is fully exerted. Mostly it is solved mathematically first and the meaning of the formula is given afterwards. Because the goal isn't clarified, it has the possibility to miss some of the important facts which appear through trial and error. If the idea is shared beforehand among other researchers, new more efficient structures might be standardized. This also suggests the problem can be introduced to the wider variety of people in different fields and more ideas would be shared. This doesn't deny the meaning of mathematical consideration. It is still necessary to find a way to be represented in a formula because the formula might notify other facts.

This time, this idea is applied to the problem of cosmic acceleration.

The main research question is whether or not the explanation done by the intuition is effective to the mystery of cosmic acceleration. In order to test its possibility, the vacuum energy is brought up, and the ideas were come up with to make an explanation. Through the interview to the specialists the validity of the ideas which was brought up by the experiential intuitions were discussed.

Cosmic acceleration was observed in 1998 [1]. Vacuum energy is one way of explaining this phenomenon. Many research projects have been done by both mathematical and observational approaches. From the mathematical approach, a problem of scale difference has been pointed out. From the observations, it is suggested that time varying model is suitable. Moreover, the results of the research by mathematics so far are said to be unnatural because they require a fine adjustment. Therefore, possibilities of different situation have been discussed.

Currently, the problems are up against the vacuum energy as a candidate of dark energy. However, by giving explanations and by refining a bit of the concept of vacuum energy, problems may be overcome. The continuity of space will be considered to the problem of the difference of the scale, . To the changing value of ω (Equation of states) rethinking of mechanism of the repulsive force is proposed.

This research is done based on interviews to the specialists who are interested in cosmic acceleration.

As a result, on one hand, the difficulty of explaining by experiential intuition was pointed out in each topic. On the other hand, there were approaches done by questioning the premise already (of the mathematics but the basics the theory relies on).

In the final analysis the approach lacked on concrete evidences and was proved untenable.

In fundamentals, basic knowledge on cosmic acceleration will be shown. In methods, researchers whom accepted the interview and the main topics will be shown. In results, questions and answers will be shown organized by the topics. In discussions, the results will be reviewed and the validity of this approach will be discussed.

2. Fundamentals

Cosmic Acceleration

“Found in 1998. Until then, it was believed that the expansion of universe will decelerate because gravity works throughout the space. However, the observational fact pointed out that there should be something which works as a repulsive force. The current physic can’t explain the phenomenon. However, because it isn’t discovered, the research of the cosmic acceleration is believed to take physic to the new place [2].”

Approaches to Cosmic Acceleration

There are mainly three types of approaches from the theoretical physics.

1. Dark energy / 2. Modified gravity theory / 3. Admitting the inhomogeneous universe [3].

1 and 2 are the approaches to revise the Einstein field equation which represents the gravitational field, the relation between the geometry of the space-time and distribution of the matter. 1 is a way introducing a new substance that has a repulsive force and 2 is a way modifying the effect of gravity in a cosmological scale. Some research suggests the time varying model of dark energy’s equation of state. 3 is the approach to revise the space principle which is a major premise in cosmology. 3 suggests the apparent cosmic acceleration.

Vacuum Energy

Vacuum Energy is one way of explaining the cosmic acceleration. It is thought as a uniformly distributed substance that work as a repulsive force. The idea is rooted in the concept of cosmological constant which was first introduced by Einstein. There are research projects that suggest the different scale problem of the vacuum energy and the dark energy, that makes it less likely to become the factor of the cosmic acceleration [4]. This essay focuses on vacuum energy because it is a quite simple concept which seems more natural then other approaches. This essay close up two features of Vacuum Energy vacuum fluctuation, and relation between the vacuum energy discussed in inflation.

Vacuum Fluctuation

Explained by uncertainty principle of Quantum mechanics, the field always has some kind of pairs of particles which appear and disappear in a very short time. Therefore, it is impossible to think of a space free from any energy. The vacuum is defined as a minimum state of the energy. This is because the appearance and the disappearance happen randomly, it is thought that there is fluctuation of the vacuum. The **Casimir effect** is the evidence of the vacuum fluctuation. It is experimented with two metal plates close together enough to create the standing wave inside both metal plates. The attractive force will be created between two metal plates due to the gap of the energy density [5].

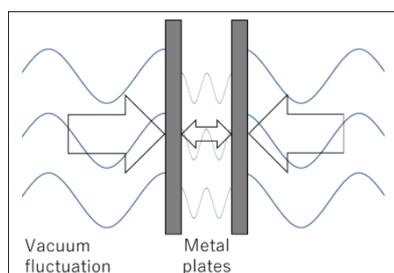


Fig. 2-1 The Casimir effect

Inflation

“According to the theory of inflation, the early universe expanded exponentially fast for a fraction of a second after the Big Bang [2].” Phase transition of vacuum is thought as a factor of inflation [6].

3. Methods

My hypothesis “Rubber band model”

In the Casimir effect, standing wave creates a gap between the fluctuation inside and outside two metal plates. If there is a different mechanism that creates the gap, understanding of the vacuum energy working as a dark energy can be changed.

This time, the idea is that, as the radiation or matter moves through the space, the fluctuation of the vacuum will get bigger. This is thought after the rubber band. When we stretch the rubber band many times, the rubber will be damaged and the strength of the contraction will decrease. The rubber band refers to the space and the force given to the rubber band refers to the movement of the radiation and the matter.

The following procedure was used to test and check the validity of the approach to explain the cosmic acceleration by explanation which is independent from the mathematical methods.

(1)Literature review

(2)Interview to the theoretical physicists

Professor Anne Davis and Doctor Philippe Brax were interviewed at the Centre for Mathematical Science (2019 July 29).

Their group studied the Cosmological Chameleon.

- The different mechanism of the created force was discussed.

Doctor Ota Atsuhisa was interviewed at the Centre for Mathematical Science (2019 July 30).

His main interest is cosmology.

- The relation between the energy of inflations

- The point scientific research should follow

were discussed.

The interview to Doctor Atsuhisa was done in Japanese. In this essay it is translated by the author.



Fig. 3-2 Professor Davis (Right) Doctor Brax (Left)

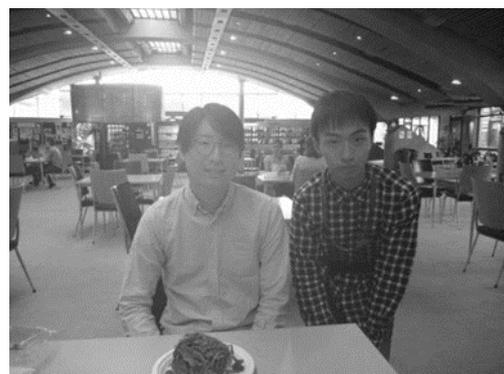


Fig. 3-1 Doctor Ota (Left)

4. Results

【Vacuum Energy】

Due to the following purposes, the following questions were asked.

- To confirm the concept of vacuum energy.

Q1 Is vacuum energy an approach from the modifying gravity or introducing a new substance?

Q2 How does uniform distribution work as a repulsive force?

Q3 How does repulsive force work at the boundary?

Q4 If Vacuum energy has a gaining force why doesn't the space contract to aim for a stable state?

※Law of increasing entropy was misunderstood and questioned as if it were talking about it will lead to the state with lower energy to be stable.

- To think of different ways of the investigation of vacuum energy.

Q5 Can the experiment of vacuum energy other than the Casimir effect be conducted in laboratories?

- To deal with the different scale of vacuum energy and dark energy.

Q6 Vacuum energy might include the potential energy but dark energy is calculated in an observed range.

- To explain where the energy comes from.

Q7 Does the law of conservation work throughout space?

Q8 Is this vacuum energy the energy from the phase transition of vacuum energy discussed in Inflation?

(Why are there no second transfer of vacuum?)

- To discuss about the different mechanism of created force by vacuum energy.

Q9 Pseudo Casimir effect

The expectations of each question are shown below

Q1 Deciding the base of the explanation and confirming the definition.

Q2 Excluding the possibility of vacuum energy not creating a repulsive force.

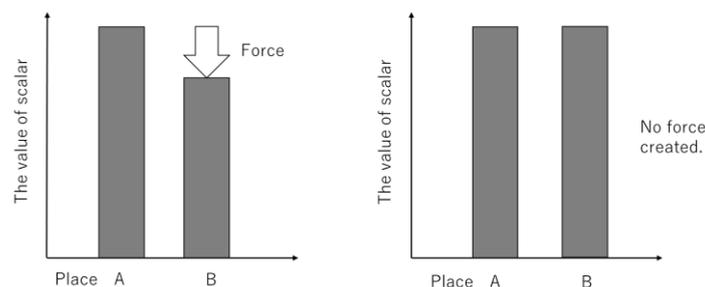


Fig. 4-1 Image of scalar and force.

Q3 Excluding the possibility of repulsive force not working as the cosmic expansion.

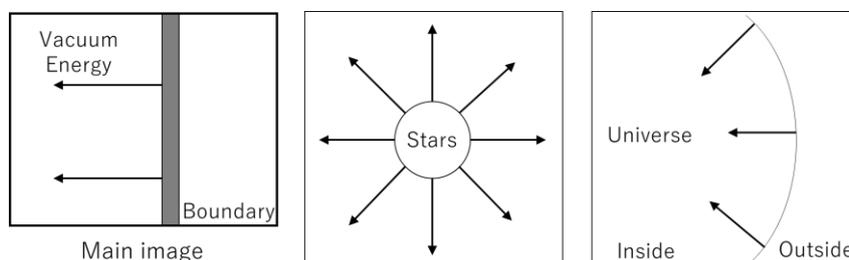


Fig. 4-2 Image of repulsive force at the boundary.

- Q4 Excluding the possibility of universe which has a nature to contract in the end.
- Q5 Finding another way to investigate vacuum energy.
- Q6 Finding a breakthrough to the problem by the explanation.

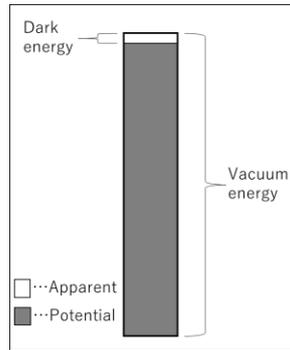


Fig. 4-3 Image of scale of vacuum energy and dark energy.

- Q7 The energy is conserved and the surplus energy is the source of vacuum energy. Inflation is in mind.
- Q8 Cosmic acceleration being explained by the second phase transition of vacuum.
- Q9 Thinking the situation from new aspect with the experiential intuitions.

Followings are the answers at the interviews. (without honorifics)

- The Concept of Vacuum Energy.

A1

Davis) “Usually, when you talk about vacuum energy, we are talking about the minimum of the potential. We are not thinking about the dark energy in sense. If we could understand vacuum energy, we might understand why the minimum is tiny and what we think it is, but you know somehow it is something that is been appearing in constant theory for a very long time, but as far as we can tell, the vacuum energy is zero.”

A2

Davis) “If it is something like a chameleon, then yes it would be uniformly distributed, but it would behave differently in different areas depending on the environment. Whereas pure vacuum energy would be independent of that of the environment. If you are talking about vacuum energy, I would say that it could be the minimum of the potential in electro weak field but it clearly isn’t because the scale is wrong.”

Brax) “The cosmology is considered as a fluid and the expansion is thought with the thermodynamics.”

A3

Brax) “You are trying to think in terms of the classical picture, where particles hitting the boundary and producing a pressure on it. I’m not sure it really works for these things. Pressure is essentially, the difference of angular momentum going on one direction. So, if you have a boundary you have a particle which hits the wall and then bounces back so the difference is that twice the value and it will give you the pressure. If you have a flow of the fluid, first you have a pressure in it, because there is no boundary, we just have most of the fluid going in one direction and nothing in the opposite direction. Cosmology is like a fluid.”

Ota) We don’t consider that there is a wall around the universe. It is because there is a space principle of homogeneous and isotropy.

A4

Davis) “No, it is more complicated than that because you got to do it in what is called a curve background, because you got to include general relativity into entropy. I mean this is what hawking think we did. Once I did classical entropy, it gets more, much more complicated in a general relativity background.”

Ota) The dark energy is thought as an energy which is likely to expand as they gain their space and energy at the same time.

- Different way of investigating vacuum energy.

A5

Davis) “There is no such thing as a true vacuum and you got tiny parts of fluctuation which breaks in an electron pair and that is what is doing. It is just phasing an electron pair in the vacuum. That is what the Casimir is measuring.”

- The different scale problem.

A6

Brax) “We don’t know how to calculate vacuum energy because our expression theoretically, they are infinites when you do the calculations. Vacuum energy depends on really short scales, that means we wouldn’t know physics up to infinitely small scales.”

Me) Bigger than plank?

Brax) “But we don’t know plank, we’ve never seen it, so theoretically, we would be able to understand at old scales but it’s not the case in extremely small scales, we do not know what is going on. So, we use formulae which are derived from what we know, and the regime we are not allowed to use it because it is too big. We get something which is infinite but physics is not infinite. It just reflects the fact that we have uncertain idea how to calculate it and primitive way of calculating that we have. It is for the future generation to aim better.

We have to be sure that we know the right string theory and whatever calculation to infinitely short distances and if we need that we could proper calculation get the fact and we prepare it to experiments. We can’t do that now because we don’t know what happens of energies higher than LHC’s. We cannot be guesswork.”

- The source of vacuum energy.

A7

Davis) “Yes.”

A8

Ota) The inflation is explained by the energy of scalar field which is called inflaton. The thing which is sure about the inflaton is that as far as there is inflaton in the field, inflation will last. Inflaton may breakup into varieties of particles but what they change into is not well known yet. When we compare the inflaton and the vacuum energy, it is similar on the formula, but the energy scale is different.

- Different mechanism of created force.

A9

Davis) “The Casimir effect comes from the vacuum fluctuation.

The vacuum energy is discussed in the cosmic acceleration as a cosmological constant. Not as a vacuum fluctuation.

Weak Casimir effect would be a bit different because it is coming from the fluctuation between two plates and it is very close together. All we know is our horizon volume. we’ve no idea what is inside the cosmos and outside the cosmos it is just we can’t imagine, can’t observe. I mean there may be an outside and it is science fiction.

What one can use to Casimir effect to try to detect any modifications of general relativity and that is something we've been trying to do."

【The Validity of the Approach】

Through the discussion with Doctor Ota Atsuhisa, the problem of this research work was pointed out.

About physics

Ota) Physic is based on the fundamental principles. The formulas are understood physically when it is derived mathematically from the fundamental principles and given the interpretation to it. We can explain the phenomenon by words, because we understand the formulas. So, just by copying the information which was written by someone else, it doesn't mean that the topic is fully understood and could be discussed. The theory of physics such as dark energy, extra detentions, and super string theory is an exciting idea which seems to be bizarre. But why it is accepted is because they are based on the concrete physical laws. There is no royal road to learning physics. If one is interested in this area, one should take the right path to study from the beginning in order to understand what is behind each idea.

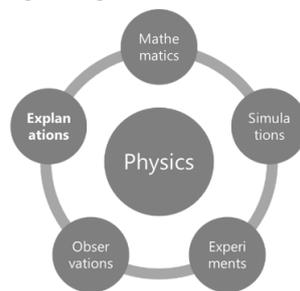


Fig. 4-4 Examples of other confirmed approaches in physics and relations between the explanations and others. (The correctness can be confirmed using other methods.)

5. Discussions

【Vacuum energy】

- The concept of vacuum energy.

First of all, the vacuum energy was out of consideration because the serious problem, the difference of the scale, was pointed out. However, there was the problem of methods of scale calculation which leaves the chance of vacuum energy as a dark energy. Although, even if the vacuum energy is taken in to the consideration, whether vacuum energy is an approach from modifying gravity or thinking of a substance depends on how the Einstein field equation is rearranged. In other words, if the equation could adopt the current situation, it really doesn't matter how the equation is interpreted because physics itself doesn't really focus on explaining phenomenon by our experiential intuitions [A1]. The uniform distribution of vacuum energy is one of the idealizations of the universe. In the real situation, the uniform distribution doesn't always mean the uniform state or work in different places, and the cosmological chameleon is the theory which the difference of different environment is taken in to consideration. [A2] There is no such thing as a boundary around the universe because there is a space principle. Universe is thought as a fluid in thermodynamics so it is not focusing on the specific particle and observing how it moves but considering the flow of universe itself from broad perspective [A2] & [A3]. Dark energy itself is considered as a substance which has a nature to expand the space and gain energy. There is some room for investigation on adopting the entropy to the curve background [A4].

- Different way of investigating vacuum energy.

The vacuum energy is a theoretical thing and hard to conduct an investigation in a laboratory free from other effects. Moreover, if the effect of vacuum energy effects is larger, the value of the experiment should appear in the investigation already [A5].

- The different scale problem.

The calculation of the infinite small scale is not yet discovered. The calculation should be done before holding an experiment so, there needs to be a break through to the problem. The explanation doesn't give any evidence for it so it will turn untenable afterwards [A6](#).

- The source of vacuum energy.

The particle in the inflation is called inflaton and the scale is different from the vacuum energy. There are several concepts that has the name of vacuum in cosmology. Before taking them into consideration, the difference between each concept needs to be understood by the core [A7](#) & [A8](#).

- Different mechanism of created force.

The force created by the gap of vacuum fluctuation is different from what vacuum energy really means. It is not likely to behave as a dark energy because the idea contradicts with the space principle of homogeneity and isotropy. It is a problem because many theories roots in space principle [A9](#).

In conclusion, vacuum energy is hard to adopt the current situation of cosmic acceleration.

【Reflections】

Through the interviews, features of experiential intuitions appeared. The experiential intuitions were based on classical physics. Current physics are thought with physics which overcame the weakness of classical physics and able to discuss about the advanced topics. This time, most of the ideas could be denied with understanding physics further.

Misunderstandings were found through the interview. Followings are the examples.

- The lack of fundamental knowledge. For example, the thermodynamics [A2](#).
- The cosmos was thought of as fluid [A3](#).
- Law of increasing entropy was thought as a phenomenon which things moves to a stable state. Aim for a lower energy [A4](#).
- Vacuum energy as a dark energy is different from the phenomenon "vacuum fluctuation" [A9](#).
- The overlooking of the space principle, homogeneity and isotropy [A9](#).
- The space expands under same condition macroscopically [A9](#).

These mistakes came from the shallow understanding of mathematics and physics. Especially, the way physics was learned was by the written explanation of the formula and the essential part of mathematics was not fully understood. Because the mathematics were neglected, the aspect of having consistency with the previous researches were screened.

The purpose of this essay was to try to explain the phenomenon with the experiential intuition. However, it turned out to be that the ideas which are not supported by mathematical evidences, have a problem in its disputability, to check its correctness, as a scientific approach. Also, it is hard to think of the consistency with the discovered theories.

As a result, the greatness of mathematics stood out as a logical and scientific approach to explain physics. It has the strength in its disputability and augmenting theories with much less risks of the contradiction with the existing theory.

Overall Discussion on the methodology.

In the field of theoretical physics, the value of the mass density of dark energy, constrained by the observational facts, was important and what it really means was not important at the first place. After we had a concrete value, we are able to talk about what it really means. Without the value, any ideas which can be objected with fundamental knowledge will also took

into consideration and it will be troublesome to talk about them. At first, verbalizing the phenomenon was expected to show the path before the confirmation by mathematics and be able to carry out the investigation more smoothly, but it seems more likely to confuse the discussion. What makes it confusing is the ambiguity that the word has. It is hard to find what is wrong when we only use the words. Moreover, the different usage of the word will have different meaning upon each people who investigates it.

Mathematics is the best way to show the correctness compared to other concept to think logically, but there might be a limitation to deal with the mystery of the cosmic. The limitation comes from the limited source which humans are capable with. At least, there might be problems which couldn't be thought of without the undiscovered mathematical theory. The mathematics are founded methods to think logically. Now researchers are thinking inside the mathematics which might be narrowing their ideas. If there will be new methods to deal with the scientific problems more freely, there are possibilities of introducing the new comers to this field. However, this essay is suggesting that it is extremely hard to construct the logical system which is superior to the mathematics. The methods need to have a same meaning to everyone and should be able to discuss its correctness or validity.

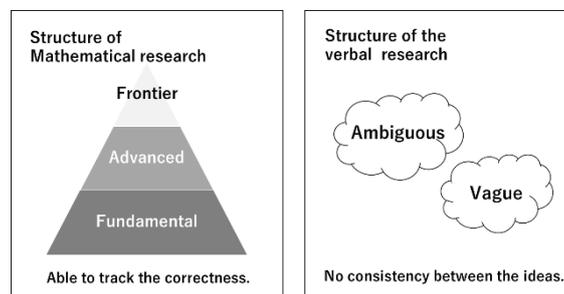


Fig. 5-1 Difference between the Mathematical research and verbal research.

6. Conclusions

The main research question was whether or not the explanation done by the intuition is effective to the mystery of cosmic acceleration. As a result, there were two main discoveries. First, most of the experiential intuition were based on classical physics. The universe is currently discussed with advanced physics so the idea based on classical physics is hard to adopt the situation. Second, the idea which was made with the combination of the concepts by the easy explanations were meaningless to be discussed. The information written in the books and papers were supported by extensive evidences. Without understanding its fundamental knowledge, it is probably impossible to discuss the advanced topics, because most of the ideas could be denied with this fundamental knowledge. Also, there were misunderstandings of the concepts due to the author's superficial knowledge. The author should study the basics more before talking about the advanced topics.

From the aspect of scientific approach, it is important that the idea is disputable. This is the problem which most of the bizarre ideas face. Through the research, the way science has developed with the logical methods were realized.

7. Future

For the problem of the calculation of the scale of vacuum energy, breakthrough is required to discuss the real possibilities of vacuum energy as a dark energy.

8. Acknowledgements

I would like to thank professor Anne Davis, Doctor Phillippe Brax, and Doctor Ota Atsuhisa who kindly accepted my interview. I also want to thank Doctor David Skinner and Doctor Enrico Pajer who introduced me such wonderful researchers.

9. References and bibliography

References

- [1] Riess, A., et al. Observational evidence from supernovae for an accelerating universe and a cosmological constant. *The Astronomical Journal*. 1998, vol. 116. The American Astronomical Society P.1009-1038 [accessed 2020/1/21]. [pdf]. Available at: <https://iopscience.iop.org/article/10.1086/300499/pdf>.
- [2] The Stephen Hawking Centre for Theoretical Cosmology. Glossary. [Accessed 2020/1/21]. [online]. Available at: <http://www.ctc.cam.ac.uk/footer/glossary.php>.
- [3] 真貝寿明『現代物理学が描く宇宙論』Tokyo, Japan:共立出版, 2018.
- [4] 須藤靖「宇宙のダークエネルギー」Japan:理論天文学宇宙物理学懇談会, 2006. P.1-58. [Accessed 2020/1/21]. [pdf]. Available at: <http://rironkon.jp/sympo06/proceedings/y.suto.pdf>.
- [5] 長島順清「真空の性質」Japan:長島順清, 2009. P.1-2, P.4-45. [Accessed 2020/1/21]. [pdf]. Available at: http://osksn2.hep.sci.osaka-u.ac.jp/~naga/kogi/shimane-class09/shimanelec2_vac.pdf.
- [6] 天文学辞典「真空の相転移」2018. [Accessed 2020/1/21]. [online]. Available at: <http://astro-dic.jp/phase-transition-of-vacuum/>.

Bibliography

(Japanese sources in Japanese syllabary order.)

- Kohler, S. (2019). Supernovae, Dark Energy, and the Fate of Our Universe. [online] AAS Nova. Available at: <https://aasnova.org/2019/04/05/supernovae-dark-energy-and-the-fate-of-our-universe/> [Accessed 21 Jan. 2020].
- Physicsworld, (2000). *Quintessence*. [online] Available at: <https://physicsworld.com/a/quintessence/> [Accessed 19 Jun. 2019].
- Riess, A.G. et al, (1998). 'Observational evidence from supernovae for an accelerating universe and a cosmological constant', *The Astronomical Journal*, [pdf] 116, pp.1009-1038. Available at: <https://iopscience.iop.org/article/10.1086/300499/pdf> [Accessed 21 Jan. 2020].
- The Stephen Hawking Centre for Theoretical Cosmology. *Glossary*. [online]. Available at: <http://www.ctc.cam.ac.uk/footer/glossary.php> [Accessed 21 Jan. 2020].
- 安東正樹 (2016)『重力波とはなにか「時空のさざなみ」が拓く新たな宇宙論』講談社 pp.13-74.
- オデンワルド, S. (2000)『宇宙 300 の大疑問』(塩原通緒訳) 講談社
- 斎藤遼 (2018)「宇宙を支配する重力法則は何か？」1st ed. [pdf] Japan:天文・天体物理 若手の会 pp.1-62. Available at: www.astro-wakate.org/ss2018/web/file/夏の学校_RyoSaito.pdf [Accessed 21 Jan. 2020].
- 須藤靖 (2002)「宇宙定数」1st ed. [pdf] Japan:須藤靖, pp.1-53. Available at: <http://www.utap.phys.s.u-tokyo.ac.jp/~suto/myresearch/lambda02.pdf> [Accessed 21 Jan. 2020].
- 須藤靖 (2006)「宇宙のダークエネルギー」1st ed. [pdf] Japan:理論天文学宇宙物理学懇談会, pp.1-58. Available at: <http://rironkon.jp/sympo06/proceedings/y.suto.pdf> [Accessed 21 Jan. 2020].
- 辻川信二 (2014)「暗黒エネルギーと修正重力理論」*日本物理学会誌*, [pdf] 69(7), p.444. Available at: <https://www.jps.or.jp/books/gakkaishi/2014/07/69-07reviews.pdf> [Accessed 21 Jan. 2020].
- 辻川信二 (2014)「暗黒エネルギーと修正重力理論」1st ed. [pdf] Kyoto:京都産業大学, pp.1-29. Available at: http://www.cc.kyoto-su.ac.jp/project/MISC/events/CST-MISC-symposium/1411/slide/CST-MISC_141108_Tsujikawa.pdf [Accessed 21 Jan. 2020].
- 天文学辞典 (2018)「真空の相転移」 [online] Available at: <http://astro-dic.jp/phase-transition-of-vacuum/> [Accessed 21 Jan. 2020].
- 土井守・松原隆彦 (2011)『宇宙のダークエネルギー「未知なる力」の謎を解く』光文社
- 中島林彦 (2019)「変容するダークエネルギー」*日経サイエンス*, (2019年5月号), pp.34-41.
- 長島順清 (2009)「真空の性質」1st ed. [pdf] Japan:長島順清, pp.1-2, pp.4-45. Available at: http://osksn2.hep.sci.osaka-u.ac.jp/~naga/kogi/shimane-class09/shimanelec2_vac.pdf [Accessed 21 Jan. 2020].
- 馬場一晴 (2009)「拡張された重力理論におけるファントムクロッシング」1st ed. [pdf] Osaka:大阪市立大学 宇宙物理・重力 研究室, pp.1-38. Available at: http://www.sci.osaka-cu.ac.jp/phys/astrophys/colloquium09_file/Bamba2009.pdf [Accessed 21 Jan. 2020].
- 福江純 (2013)『完全独習現代の宇宙論』講談社サイエンティフィク
- 真貝寿明 (2018)『現代物理学が描く宇宙論』共立出版
- 松原隆彦 (2014)『大規模構造の宇宙論 宇宙に生まれた絶妙な多様性』共立出版
- 村山斉 (2013)『村山さん、宇宙はどこまでわかったんですか？ビッグバンからヒッグス粒子へ』朝日新書 pp.169-203.
- リオーダーン, M.・シュラム, D.N. (1994)『宇宙創造とダークマター』(青木薫訳) 吉岡書店 pp.1-38, pp.206-239.